Math 55 Quiz 5 DIS 106

Name: ______________________________ 7 Mar 2022

1. Let f_n be the n^{th} Fibonacci number. (Recall that these are defined by $f_1 = 1, f_2 = 1$, and $f_{n+2} = f_n + f_{n+1}$ for $n \geq 1$.)

   Use induction to show that $f_n < 2^n$ for every $n \geq 1$. [5 points]

   Let $P(n)$ be the proposition that $f_n < 2^n$.
   For $n = 1$, $f_1 = 1 < 2 = 2^1$, so $P(1)$ is true.
   For $n = 2$, $F_2 = 2 < 4 = 2^2$, so $P(2)$ is true.
   Suppose $P(k)$ is true for all $k \leq n$, where $n \geq 2$
   Then

$$\begin{aligned} f_{n+1} &= f_{n-1} + f_n \\ &< 2^{n-1} + 2^n \text{ by } P(n-1) \text{ and } P(n) \\ &= 2^{n-1} \cdot 3 \\ &< 2^{n-1} \cdot 4 = 2^{n+1} \end{aligned}$$

   So $P(n+1)$ is true.
   By strong induction, $P(n)$ is true for every $n \geq 1$.

2. Find the number of bit strings of length not exceeding 10 that have at least one 0 bit. Explain your answer. [5 points]

There are 2^n bit strings of length n in total, and only one of them does not contain any 0 bits (namely, 1...1). So there are exactly $2^n - 1$ bit strings of length n that have at least one 0 bit.
Hence there are $\sum_{n=1}^{10}(2^n - 1) = 2^{11} - 12 = 2036$ bit strings of length not exceeding 10 that have at least one 0 bit.